NUMERICAL SOLUTION TO THE PROBLEM OF HEAT TRANSFER IN A BED WITH PERIODIC REVERSALS OF THE HEAT CARRIER FLOW

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A numerical solution is obtained to the problem of regenerative heat transfer in a packed bed of granular material with a counterflow of heat carriers.

The heat transfer during periodic reversals of the heat carrier flow is very important in practical applications, namely in heat and mass transfer processes such as, for instance, desiccation, adsorption, extraction, etc., and in "purely" heat transfer processes as in regenerative apparatus.

Until now the mathematics of this problem has been formulated with the final value of the unknown quantity (e.g., the bed temperature) at the end of the first period (e.g., the period of heating the bed elements), assumed to be its initial value in the second period (cooling) and its final value in this period taken, in turn, as its initial value in the next period, etc. This procedure has led to serious mathematical difficulties in solving the problem and to solutions too unwieldy for practical use.

The heat transfer* during flow reversal proceeds in such a way that within a certain period of time or at a certain location along the flow of heat absorbing (or heat emitting) material one of the heat carriers penetrates the bed material in one direction, while within the next period (or at another location) the other heat carrier penetrates the bed material in the opposite direction. In processes which involve heat and mass transfer the initial parameter values of both heat carriers (their temperatures, velocities, densities, etc.) are, as a rule, identical, but in "pure" heat transfer processes they are different.

In principle, the gist of the new approach to solving the problem of heat transfer during flow reversals is that both heat carriers are regarded as a single one which periodically reverses its direction of flow, with the parameter values of such a heat carrier not necessarily the same in each direction. Such a formulation of the problem, where the interplay between the temperatures of the material during different process periods is disregarded, makes it feasible to determine the temperature field of the heat carrier at any instant of time and, subsequently, the temperature field of the bed material.

For this case, the system of equations describing the heat transfer in the bed during zero-gradient heating can be written in dimensionless form as follows [1]:

$$\psi(z)\frac{\partial T(y,\ z)}{\partial y}+k\frac{\partial T(y,\ z)}{\partial z}=-[T(y,\ z)-\Theta(y,\ z)], \tag{1}$$

$$\frac{\partial\Theta(y,\ z)}{\partial z}=T(y,\ z)-\Theta(y,\ z), \tag{2}$$

the boundary condition as

$$T[y(z),\ z]=T_{\mathrm{H}}\cos\frac{\pi}{2y_{\mathrm{f}}}y(z)+T_{\mathrm{C}}\sin\frac{\pi}{2y_{\mathrm{f}}}y(z), \tag{3}$$

*Inasmuch as the new approach to this problem will be based mainly on a formulation of the boundary conditions, it is immaterial whether it concerns heat transfer or heat and mass transfer or any other mode of transmitting energy and matter.

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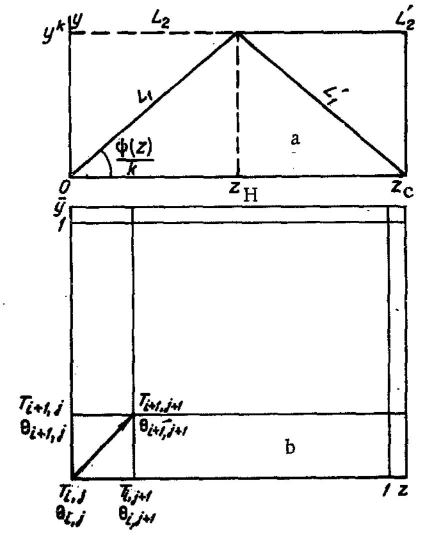

Fig. 1. Concerning the numerical method of solving the problem: (a) characteristics, (b) computer-aided calculation. Heating period (o–z_H), cooling period z_H–z_C.

and the initial condition as

$$T(y,\ 0)=e^{-y}. \tag{4}$$

System (1), (2) differs from the well known equations in [2] by the factor $\psi(z)$, while the boundary condition (3) is essentially new. It has been assumed in the derivation of (1), (2) that the initial velocity of the heat carrier (before entering the bed) is the same in both directions. This stipulation is not a matter of principle, inasmuch as it only simplifies the original equations (1) and (2) without making this analysis less general but, at the same time, reflecting a situation prevalent in most practical cases. Let us, however, dwell on this matter somewhat longer. The dimensionless space coordinate $y = x\alpha F/c_p vm$ and the dimensionless time coordinate $z = \tau\alpha F/c_M(1-m)$ both contain quantities α, c_p, and v, which generally have different values during the heating period and during the cooling period or, in other words, have different values for different heat carriers. Nevertheless, in order to simplify the calculations and the subsequent analysis, we will henceforth always assume the same values for y and z. The values of the various parameters of the two heat carriers at equal y and z correspond, therefore, to different x and τ coordinates during heating and cooling, i. e., at the same y and z we consider the temperatures of the heat carriers (of the heating and of the cooling medium) and of the packing at various bed sections and at different instants of time, the latter counted from the start of the process. As a practical matter, this will be reflected in subsequent calculations by the stipulation that $y_H \neq y_C$ at the same location x and $z_H \neq z_C$ at the same time τ.

We now introduce the ratio $u = z_C/z_H$ of dimensionless cooling time to dimensionless heating time of a dispersion bed (in the case of a regenerative heat transfer) so that the periodic step function $\psi(z)$ can be expressed as

$$\psi(z)=\begin{cases}+1, & z_H(n+n\mu)\leqslant z<z_H[(n+1)+n\mu],\\ -1, & z_H[(n+1)+n\mu]\leqslant z<z_H[(n+1)(1+\mu)]\end{cases} \tag{5}$$
$$(n=0,\ 1,\ 2,\ 3\ \ldots).$$

Analogously, function y(z) can be written as

$$y(z)=\begin{cases}0, & z_H(n+n\mu)\leqslant z<z_H[(n+1)+n\mu],\\ y_f, & z_H[(n+1)+n\mu]\leqslant z<z_H[(n+1)(1+\mu)]\end{cases} \tag{6}$$
$$(n=0,\ 1,\ 2,\ 3\ \ldots).$$

Equations (1)-(6) completely describe the heat transfer in a bed of disperse material during periodic reversals of the heat carrier flow and zerogradient heating of the bed elements. An analytical solution of system (1)-(6) involves serious mathematical difficulties. However, if we forego a complete theoretical analysis of the effects of various parameters on the process trend and, instead, consider only determining the temperature field of the granular bed and of the heat carrier (gas and air), then a computer-aided numerical solution of the problem will be entirely adequate.

For this purpose, then, we rewrite Eq. (1) as

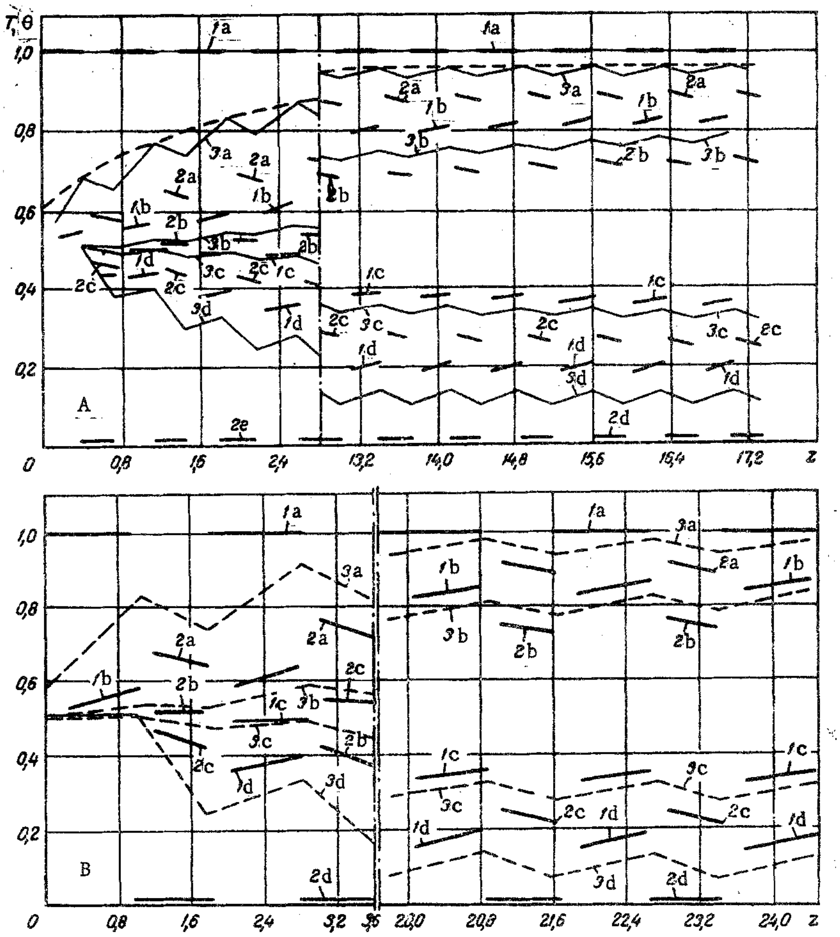

Fig. 2. Temperature T of the gas (1), of the air (2), and temperature Θ of the packing elements, as functions of the dimensionless time z at: (A) $z_c = 0.722$, $z_H = 0.417$, $z_C = 0.305$, $\mu = 0.732$, and full bed height $y_H^f = 34$, $y_C^f = 30$, (B) $z_c = 1.8$, $z_H = 1.04$, $z_C = 0.76$, $\mu = 0.924$ and full bed height $y_H^f = 36.5$, $y_C^f = 31.0$. For (A) at section $y_H = y_C = 0$ (a), $y_H = 3.4$ and $y_C = 3.0$ (b), $y_H = 30.6$ and $y_C = 27.0$ (c), $y_H = 34$ and $y_C = 30$ (d). For (B) at section $y_H = y_C = 0$ (a), $y_H = 3.65$ and $y_C = 3.1$ (b), $y_H = 32.85$ and $y_C = 27.9$ (c), $y_H = 36.5$ and $y_C = 31.0$ (d).

$$\frac{\partial T(y,\ z)}{\partial z} + \frac{\psi(z)}{k} \cdot \frac{\partial T(y,\ z)}{\partial y} = -\frac{1}{k}\,[T(y,\ z) - \Theta(y,\ z)]. \tag{7}$$

Through every point of the region there pass two characteristics, L_1 and L_2, with the slopes $\psi(z)/k$ and 0 respectively to the z-axis (Fig. 1a). For a numerical solution it is convenient to make the following change of variables:

$$y = y_f \bar{y}, \text{ where } 0 \leqslant y \leqslant y_f,\quad 0 \leqslant \bar{y} \leqslant 1,$$

$$z = z_H \bar{z}, \text{ where } 0 \leqslant z \leqslant z_H,\quad 0 \leqslant \bar{z} \leqslant 1.$$

Taking this into account, we rewrite Eqs. (2) and (7) as

$$\frac{\partial T(\bar{y},\ \bar{z})}{\partial \bar{z}} + \frac{\psi(\bar{z})\, z_H}{k y_f} \cdot \frac{\partial T(\bar{y},\ \bar{z})}{\partial \bar{y}} = -\frac{z_H}{k}\,[T(\bar{y},\ \bar{z}) - \Theta(\bar{y},\ \bar{z})], \tag{8}$$

$$\frac{\partial \Theta(\bar{y},\ \bar{z})}{\partial \bar{z}} = z_H\,[T(\bar{y},\ \bar{z}) - \Theta(\bar{y},\ \bar{z})]. \tag{9}$$

According to Fig. 1a, $|\psi(\bar{z}) z_H / k y_f| = 1$, i.e., after the change of variables characteristic L_1 will be inclined to the z-axis at a 45° angle.

TABLE 1. Values of the Final Average-Over-the-Period Gas and Air Temperature

Dimensionless time				Length of cycle z_c	Dimensionless bed height		Final average-over-the-period temperatures	
of heating		of cooling						
start z_H^0	end z_H^f	start z_C^0	end z_C^f		for heating y_H	for cooling y_C	during heating $\bar{T}_H$	during cooling $\bar{T}_C$
10,108	10,525	10,525	10,83	0 722	62,0	52,0	0,214	0,854
16,464	17,07	17,07	17,64	1,176	60,0	60,0	0,170	0,879
15,3	26,34	26,34	27,1	1,803	60,9	51,6	0,163	0,916

Thus, instead of (8), we have now

$$\frac{\partial T(\bar{y}, \bar{z})}{\partial z} + \psi(\bar{z}) \frac{\partial T(\bar{y}, \bar{z})}{\partial \bar{y}} = -\frac{z_H}{k}[T(\bar{y}, \bar{z}) - \Theta(\bar{y}, \bar{z})]. \tag{10}$$

We will now consider the heating period. The differentials of functions T and Θ for Eqs. (10) and (9) along curves L_1 and L_2 are respectively equal to [3]

$$dT_i = -\frac{z_H}{k}(T_i - \Theta_i)h_f = -y_K(T_i - \Theta_i)h_f,$$

$$d\Theta_i = z_H(T_i - \Theta_i)h_f,$$

inasmuch as $y_f = z/k$ because $\psi(z) = +1$ during the heating period. Since y and $\bar{z}$ vary within the range $0 \leq \bar{y}$ or $\bar{z} \leq 1$ when $0 \leq \bar{y} \leq y_f$ and $0 \leq \bar{z} \leq z_H$ respectively, hence $\bar{y}$ and $\bar{z}$ can be subdivided into equal numbers of intervals (Fig. 1b). In the resulting grid j denotes the number of a vertical strip and i denotes the number of a horizontal strip, with i = 0, 1, 2, ... and j = 0, 1, 2, ... representing respectively equidistant values of arguments $\bar{y}$ and $\bar{z}$ at an interval h. Knowing the values of T and 0 along the j-th vertical, we compute their values in the direction of the characteristics along j + 1, etc. until the value $z_1 = z_1/z_H$ has been reached. Since the last strip before z_H does not exactly hit point z_H, we extrapolate to point z_H on the basis of the last two strips.

Computations are made according to the Euler method with the following conversions [4]:

$$dT_i = -(T_{i,j} - \Theta_{i,j})\, y_f h_{y,z},$$

$$T_i = T_{i,j} - (T_{i,j} - \Theta_{i,j})\, y_f h_{y,z},$$

$$d\Theta_i = (T_{i+1,j} - \Theta_{i+1,j})\, z_H h_{y,z},$$

$$\Theta_i = \Theta_{i+1,j} + (T_{i+1,j} - \Theta_{i+1,j})\, z_H h_{y,z},$$

$$dT_i' = -(T_i - \Theta_i)\, y_f h_{y,z},$$

$$d\Theta_i' = (T_i - \Theta_i)\, z_H h_{y,z},$$

$$T_{i+1,j+1} = T_{i,j} + \frac{dT_i + dT_i'}{2},$$

$$\Theta_{i+1,j+1} = \Theta_{i+1,j} + \frac{d\Theta_i + d\Theta_i'}{2}.$$

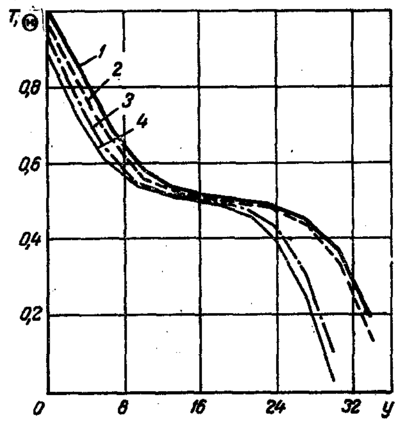

Fig. 3. Temperature profile Θ of the packing elements and T of the gas and the air, across the bed height y after 24 cycles of dimensionless heating time z = 17.02 and dimensionless cooling time z = 17.33 (process parameters the same as in Fig. 2A): heating T (1) and Θ (2), cooling T (3) and Θ (4).

At $y = 0$ one computes only Θ with the value of T given; at the initial time ($z = 0$) the values of both T and Θ are given.

For the cooling period, when $\psi(z) = -1$, the computations are analogous.

On the basis of the preceding relations, the authors have used a model M-220 computer for determining the temperature field of a disperse packing and of the heat carrier in time z and coordinate y (bed height). The computations were made for the regenerative mode of heat transfer during a counterflow of two heat carriers in a rotary heat exchanger, with two cycles per rotor revolution (one cycle consisting of a heating and a cooling period), the basic parameters varied over the following ranges: dimensionless initial temperature of the heat carrier $T_H = 1$, dimensionless initial temperature of the cooling air $T_C = 0.018$, dimensionless bed height for heating $y_H = 32$-62 and for cooling $y_C = 32$-52, dimensionless cycle time $z_c = 0.235$-1.8, dimensionless heating time $z_H = 0.117$-1.04, dimensionless cooling time $z_C = 0.117$-0.763, rotor speed $n = 1.0$-7.5 rpm, the heating and the cooling periods $\tau_H = \tau_C = 1.5$-11.25 sec.

Some results of the computations are shown graphically in Figs 2 and 3 in the form of relations $T = f(z)$, $\Theta = f(z)$, $T = f(y)$, and $\Theta = f(y)$.

In practical terms, most significant is the temperature as a function of time for the end sections of a bed, inasmuch as the temperature trend there characterizes the thermal efficiency of the apparatus. For this reason, in Fig. 2 are also shown temperature curves for the entrance section and the exit section of a bed. According to the diagram, the initial gas temperature (at $y = 0$) and air temperature (at $y = y_C^f$) are constant quantities (straight lines 1a, 2d) with respect to the boundary conditions; the temperature of bed components increases during every cycle from the very start of the heat transfer process within the initial bed segments (Fig. 2, curve 3a), while the packing temperature rises during the heating periods and naturally drops during the cooling periods. Of all the curves shown in Fig. 2, the largest number of cycles was computed for curves in Fig. 2A. In the last cycle here ($z = 16.6$-17.02, curve 3a) the packing temperature at section $y = 0$ is highest, approximately $\bar{\Theta}_H^0 = 0.954$. The lowest packing temperature of the final section ($y_H^f = 32.0$) during the heating period is $\bar{\Theta}_H^f = 0.104$ in the last cycle, while the mean temperature at this section is $\bar{\Theta}_H^f = 0.118$ and the temperature difference across the height $\Delta \bar{\Theta}_H = \bar{\Theta}_H^0 = \bar{\Theta}_H^f = 0.954 - 0.118 = 0.836$ is thus quite appreciable. At the same time, the temperature variation in every packing section is rather small within one period and a curve drawn through the test points (e. g. , the dashed line in Fig. 2A) indicates an exponential temperature change with an asymptotic approach to a constant steady-state level. At section $x = y = 0$ the air temperature also rises continuously and, toward the end of the last cycle, reaches rather high average (over the cooling cycle) levels of 0.897 (Fig. 2A, curve 2a), 0.857 (Fig. 2B, curve 2a), and 0.908. This is the final and thus also the highest air temperature. During each period, characteristically, the temperature of bed components and gases (air) varies linearly or very nearly so.

Wide fluctuations of the packing temperature during heating and cooling are noted at the end sections ($y = 0$ and $y = y_f$), where the heat transfer rate is highest. The temperature profiles of the packing, the gas, and the air across the bed height are shown comprehensively in Fig. 3. The temperature curves are steep here with respect to the y-axis for the initial bed segments, become flatter for the middle portion, and then again steeper for the final segments. With the parameter values given in Fig. 3, the heat transfer in the middle portion of the bed is still slow after 23 cycles ($z = 16.6$); in this case, evidently, the bed is too high.

In the transient state, the final temperatures of gas and hot air are appreciably affected by different cycle lengths at about the same bed height $y = 32$. For the 13th cycle at $z_c = 0.235$, for example, the average-over-the-period temperatures are $\bar{T}_H$ (gas) $= 0.341$ and $\bar{T}_C$ (air) $= 0.742$; at $z_c = 1.176$ we have $\bar{T}_H = 0.193$ and $\bar{T}_C = 0.857$; at $z_c = 1.807$ we have $\bar{T}_H = 0.172$ and $\bar{T}_C = 0.908$. Thus, while z_c changes from 0.235 to 1.803, i. e. , by a factor of 7.7, the final gas temperature changes from 0.341 to 0.172 or by 0.169 and the final air temperature changes from 0 .742 to 0.908 or by 0.166; the difference is therefore, large. If the initial gas temperature were 2000°C, for example, then, after the said extension of the cycle length, the final gas temperature would drop by 338°C and the final air temperature would rise by 332°C, i. e. , this difference would become appreciable in high-temperature processes.

An analogous pattern is noted also at large values of the dimensionless bed height, after the same number of cycles since the apparatus has started to operate. Data for the 15th cycle are given in Table 1.

With approximately the same bed height, according to the table, the gas temperature is lowered by 0.214–0.163 = 0.051 and the air temperature is raised by 0.916–0.854 = 0.062 when the cycle length is extended from 0.722 to 1.807 or by a factor of 2.5.

Consequently, an extension of the cycle length within definite limits will, with the bed height as given here, cause a drop in the final temperature of existing gas and a corresponding rise in the temperature of heated air. It is to be noted, however, that, after the same number of cycles, the heat transfer time becomes much longer during a longer cycle. After 15 cycles, for example, $z = 10.83$ with $z_c = 0.722$ and $z = 27.1$ with $z_c = 1.857$. This affects the final temperatures of heat carriers during the transient only, if the comparison is based on the same number of cycles. During the steady state, the number of regenerator cycles since the start does not affect the temperature field.

NOTATION

$y = (\alpha F/c_p vm)x$	is the dimensionless height (thickness) of disperse packing bed;
$z = (\alpha F/c_M(1-m))\tau$	is the dimensionless time;
$\Theta(y, z) = \vartheta(y, z) - \vartheta_0/t_0 - \vartheta_0$	is the dimensionless temperature of bed elements;
$T(y, z) = t(y, z) - \vartheta_0/t_0 - \vartheta_0$	is the dimensionless temperature of heat carrier;
$K = (c_p/c_M) \cdot (m/1-m)$;	
c_V	is the specific heat (on volume basis) of heat carrier, $kcal/m^3 \cdot {}^\circ C$;
c_M	is the specific heat of packing elements, $kcal/m^3 \cdot {}^\circ C$;
F	is the specific surface of particles per unit bed volume, m^2/m^3;
m	is the bed porosity, m^2/m^2;
t	is the instantaneous temperature of heat carrier, °C;
ϑ	is the instantaneous temperature of bed elements, °C;
x	is the space coordinate along fluid flow;
v	is the true filtration velocity of heat carrier through bed, m/h;
α	is the coefficient of heat transfer from heat carrier to surface of bed particles, $kcal/m^2 \cdot h \cdot {}^\circ C$;
τ	is the time, h.

Subscripts

H refers to heating;
C refers to cooling;
c refers to cycle.

Superscripts:

o refers to initial value;
f refers to final value.

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